

NASA TECHNICAL NOTE



NASA TN D-2585

NASA TN D-2585

PACILITY FORM 808

N65 14559
(ACCESSION NUMBER)

27
(PAGES)

(NASA CR OR TMX OR AD NUMBER)

(THRU) _____

(CODE) 1

(CATEGORY) 12

GPO PRICE \$ _____

OTS PRICE(S) \$ 1.00

Hard copy (HC) _____

Microfiche (MF) .50

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Office of Technical Services, Department of Commerce,
Washington, D.C. 20230 -- Price \$1.00

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SUMMARY

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A simple one-dimensional model is postulated for the problem of pressurizing a cylindrical tank containing a liquefied gas during outflow. The fluid-dynamic and heat-transfer equations are put into a form convenient for a numerical approximation. Finite difference approximations to the equations are given, and the details of a numerical solution are explained. Axial temperature distributions in the ullage gas and the tank wall are a part of the solution, as well as the amount of pressurizing gas required.

Results are obtained on an IBM 7094-II computer for 19 liquid-hydrogen examples for which experimental data are available for comparison. These cover a wide range of tank pressures, outlet flow rates, and pressurizing gas temperatures and include helium and hydrogen as pressurizing gases. A comparison of calculated with experimental pressurant-mass requirements is made, and, where practicable, gas and wall axial temperature distributions are also compared with the data. The agreement between the calculated and experimental results is good. The average calculation time was about 24 seconds per problem.

Some additional calculations are made to check the effects of (1) using a perfect gas equation of state in the calculations, (2) failing to include heat transfer to internal hardware, and (3) using a heat-transfer coefficient computed from a standard free-convection formula.

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INTRODUCTION

Liquefied gases, such as oxygen, hydrogen, and fluorine are in current use or are being contemplated for use as rocket propellants. When such liquids are expelled from a tank by the admission of a pressurizing gas, a complicated pattern of heat and mass transfer occurs. The ullage gas exchanges heat with both the tank walls and the liquid. Mass transfer may occur at the liquid surface or from the wetted tank walls. Extreme temperatures and temperature changes occur, and these are accompanied by large variations in thermal properties of the tank wall material. Sloshing of the liquid and a variation in the effective gravity field, which may also be encountered, further complicate the problem.

Liquid hydrogen, because of its very low temperature, involves the most severe thermal problems. An investigation of these problems is hampered by the fact that accurate detailed experimental data inside a discharging hydrogen tank are difficult to obtain. The systematic experiments so far conducted (e.g., refs. 1 and 2) have employed relatively small tanks. Although some detailed data may be expected from larger tanks in the future, such data may never be extensive.

It is important, under the circumstances, to proceed analytically toward a better understanding and prediction of internal tank phenomena such as temperature distribution in the ullage gas. A start can be made with an analysis that excludes such complicating factors as sloshing and zero gravity fields, although these will be important in some applications. Such an analysis is given by Arpaci and Clark in reference 3 and related papers. The analysis provides a means for predicting pressurant requirements and gas-to-wall heat transfer, as well as temperature distributions in the gas and in the wall. Thus, the effects on these quantities of a variety of parameters could be investigated analytically. A number of assumptions, however, are made in the analysis that somewhat limit its applicability. In order to maintain linearity of the differential equations involved, the authors restrict the analysis to problems having constant outflow rate, constant tank pressure, and constant initial gas and wall temperatures. It was necessary also to assume that the gas density and velocity, the gas-to-wall heat-transfer coefficient, and the gas and wall specific heats are constant.

Since the assumptions of reference 3 appear to be somewhat restrictive, it was felt that, for use with hydrogen, at least, a more general analysis should be developed. In doing this, an effort was made to retain all the characteristics of the problem, such as variable gas density and variable tank wall specific heat, that seemed likely to influence the results significantly for liquid hydrogen. No attempt was made to keep the equations linear, but an effort was made to keep the calculation time down.

The resulting equations are complex, and a numerical solution is clearly indicated. The equations for the numerical solution are developed, and the procedure for solving them is given in detail. Finally, a comparison of analytical and experimental results is made by using cylindrical tank data from the experiments described in references 1 and 2. These comparisons cover a wide range of tank pressures, inlet gas temperatures, and liquid outflow rates. Since the present work was begun, another analysis has appeared. This analysis (ref. 4) includes features not present in the one to be described here; for example, thermal conduction in the gas and wall, and diffusion of one ullage gas into another. The assumption of constant radial and circumferential values of temperature and velocity is made in both analyses. Although the method of the present report is less comprehensive than the method of reference 4, it is correspondingly simpler and faster to execute (20 to 40 sec per problem). In addition, the numerical methods used in the two approaches are quite different. For these reasons, it is felt that the present analysis is also of interest.

ANALYSIS

Consider the tank, cylindrical except for the ends, shown schematically in figure 1. The tank is partially filled with liquid, and the remainder of the volume (ullage) contains a single-component pressurizing gas. As liquid is withdrawn from the bottom of the tank at a prescribed rate, gas is added to the ullage at a rate sufficient to maintain the pressure required at each instant.

Only the cylindrical section of the ullage space (i.e., the part from $x = 0$ to $x = l(t)$ in fig. 1) is considered in this analysis. Accordingly, the terms "inlet", "inlet gas temperature", and "inlet velocity" will be used to refer to conditions at $x = 0$.

The analysis covers the period of time beginning with the start of outflow. Therefore, time $t = 0$ occurs at the start of outflow. "Initial conditions" are the conditions at time $t = 0$.

Assumptions

The following assumptions are made in an attempt to simplify the analysis and the subsequent numerical solution while retaining the essential features of the problem:

(1) The ullage gas is nonviscous.

(2) The ullage gas velocity is everywhere parallel to the tank axis and does not vary radially or circumferentially.

(3) The tank pressure does not vary spatially.

(4) The ullage gas temperature does not vary radially or circumferentially.

(5) The tank wall temperature does not vary radially or circumferentially.

(6) No heat is transferred axially in either the gas or the wall.

(7) No condensation or evaporation occurs.

(8) No heat is transferred from the gas to the liquid.

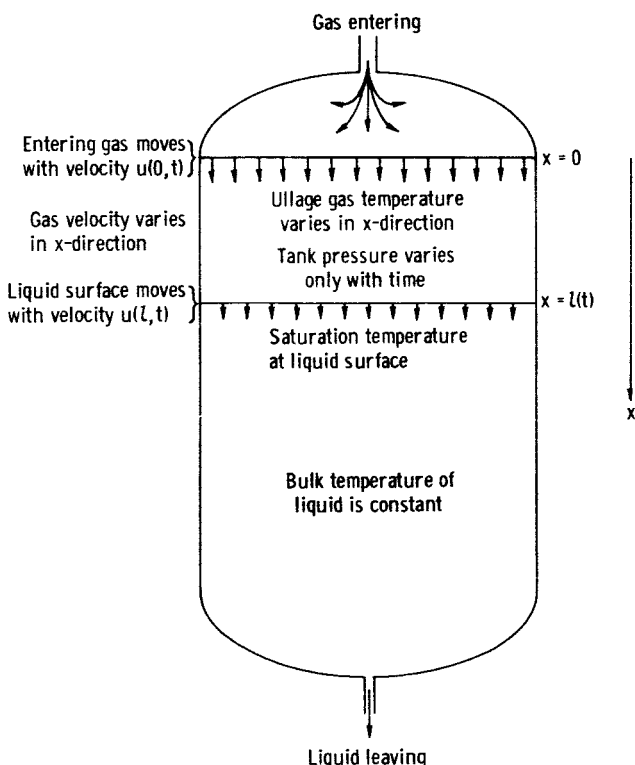


Figure 1. - Schematic drawing of cylindrical tank.

With these assumptions the problem is reduced to a one-dimensional, non-steady, nonviscous-flow problem with heat addition in the ullage gas.

Assumptions (1) and (2) lead to the following simple flow model. The pressurizing gas enters the tank uniformly at $x = 0$ (fig. 1) with velocity $u(0,t)$ and proceeds downward in the tank with a velocity $u(x,t)$ that varies with time and axial location but not with radial or circumferential location. That is, no mixing of the ullage gas takes place.

Assumption (3) is not very restrictive in rocket applications. The small changes in momentum of the gas from top to bottom of the ullage require negligible pressure gradients. Also, because of the low gas density, the pressure gradient due to vehicle acceleration is generally small.

Assumption (4) arises from observation of data obtained at the Lewis Research Center in the experiments described in reference 1. Later experiments at Lewis (performed by Richard DeWitt and others) with a variety of inlet gas diffusers have substantiated the assumption. Only when a straight inlet pipe, directed vertically downward, was used did any appreciable radial temperature gradient occur. All the data on which the assumption is based, however, were obtained from cylindrical tanks having a low heat leak. The fluid-flow and heat-transfer processes that give rise to the nearly radially constant temperatures are not understood. Nevertheless, the empirical fact can be used, and the resulting simplification of the problem is very great.

Assumption (5) is justified by the high thermal conductivity and the thinness of the metal tank wall.

Assumption (6) arises from the low thermal conductivity of the ullage gas and the relative thinness of the metal tank wall.

Assumption (7) appears to be justified by the data of references 1 and 2. The tests conducted, however, cannot be considered conclusive on this point. Future research, especially on larger tanks, may require this assumption to be changed.

Assumption (8) is based on the low thermal conductivity of the ullage gas.

Equations

The basic equations required are the first law of thermodynamics, the continuity equation, and the equation of state for a real gas. The form of the equations used here can be found in reference 5. The momentum equation is not needed since no spatial pressure change is considered (assumption (3)).

First law of thermodynamics. - Applied to a fluid particle in a reversible process, the first law of thermodynamics can be written (ref. 5, p. 189)

$$\frac{De}{Dt} + \frac{P}{J} \frac{D}{Dt} \left(\frac{1}{\rho} \right) = Q \quad (1)$$

The specific heat-transfer rate Q is defined as the rate at which heat (positive or negative) is being added to the particle per unit mass. (All symbols are defined in the appendix.) The derivatives appearing in equation (1) are defined by

$$\frac{Df(x,t)}{Dt} \equiv \frac{\partial f(x,t)}{\partial t} + u(x,t) \frac{\partial f(x,t)}{\partial x} \quad (2)$$

where $f(x,t)$ is any fluid property. Physically this derivative represents the rate at which the property $f(x,t)$ is changing as a particle is followed along its path.

The specific heat-transfer rate Q in equation (1) includes heat transfer from tank walls and also from any pipes, instruments, baffles, or other interior hardware. Since the ullage-gas temperature does not vary radially (assumption (4)), the heat flow into the gas is not assumed to take place by conduction. Instead, any heat flowing to the gas at an axial location is distributed instantly and uniformly throughout the gas at that axial location. The specific rate of heat transfer from the tank wall to the gas at a point on the x -axis is given by

$$Q_w = \frac{2\pi r h [T_w(x,t) - T(x,t)] \Delta x}{\pi r^2 \Delta x \rho} = \frac{2h [T_w(x,t) - T(x,t)]}{r \rho} \quad (3)$$

where Δx is a small increment about x as center. Similarly the specific rate of heat transfer from other interior hardware to the ullage gas is given by

$$Q_I = \frac{q_I C}{\pi r^2 \rho} \quad (4)$$

where q_I is the rate of heat transfer per unit area and C is an effective perimeter of the interior hardware.

Since $Q = Q_w + Q_I$, equations (1), (3), and (4) can be combined to give

$$\frac{De}{Dt} + \frac{P}{J} \frac{D}{Dt} \left(\frac{1}{\rho} \right) = \frac{2h(T_w - T)}{r \rho} + \frac{q_I C}{\pi r^2 \rho} \quad (5)$$

With specific enthalpy defined as

$$i \equiv e + \frac{P}{J \rho} \quad (6)$$

equation (5) becomes

$$\frac{Di}{Dt} - \frac{1}{J\rho} \frac{DP}{Dt} = \frac{2h(T_w - T)}{r\rho} + \frac{q_I C}{\pi r^2 \rho} \quad (7)$$

Since i is a function of T and P , differentiation gives

$$\frac{Di}{Dt} = \left(\frac{\partial i}{\partial T} \right)_P \frac{DT}{Dt} + \left(\frac{\partial i}{\partial P} \right)_T \frac{DP}{Dt} = c_p \frac{DT}{Dt} + \left(\frac{\partial i}{\partial P} \right)_T \frac{DP}{Dt} \quad (8)$$

Using equation (8) in equation (7) yields

$$c_p \frac{DT}{Dt} - \left[\frac{1}{J\rho} - \left(\frac{\partial i}{\partial P} \right)_T \right] \frac{DP}{Dt} = \frac{2h(T_w - T)}{r\rho} + \frac{q_I C}{\pi r^2 \rho} \quad (9)$$

It is shown in reference 5 (p. 23) that

$$\frac{1}{J\rho} - \left(\frac{\partial i}{\partial P} \right)_T = \frac{T}{J} \left[\frac{\partial}{\partial T} \left(\frac{1}{\rho} \right) \right]_P \quad (10)$$

Substituting into equation (9) from equation (10) gives

$$c_p \frac{DT}{Dt} - \frac{T}{J} \left[\frac{\partial}{\partial T} \left(\frac{1}{\rho} \right) \right]_P \frac{DP}{Dt} = \frac{2h(T_w - T)}{r\rho} + \frac{q_I C}{\pi r^2 \rho} \quad (11)$$

Equation of state. - The equation of state for a real gas is commonly written in terms of a compressibility factor $Z(P, T)$ in the form

$$MP = Z\rho RT \quad (12)$$

where M is molecular weight and R is the universal gas constant. Holding pressure constant and differentiating equation (12) result in

$$\left[\frac{\partial}{\partial T} \left(\frac{1}{\rho} \right) \right]_P = \frac{R}{MP} \left(Z + T \frac{\partial Z}{\partial T} \right) = \frac{RZ_1}{MP} \quad (13)$$

where

$$Z_1 \equiv Z + T \frac{\partial Z}{\partial T} \quad (14)$$

Substituting from equation (13) into equation (11) gives

$$c_p \frac{DT}{Dt} - \frac{RTZ_1}{JMP} \frac{DP}{Dt} = \frac{2h(T_w - T)}{r\rho} + \frac{q_I C}{\pi r^2 \rho} \quad (15)$$

From assumption (3) it follows that $\frac{\partial P}{\partial x} = 0$. Using this fact and equation (2), equation (15) becomes

$$c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) - \frac{RTZ_1}{JMP} \frac{\partial P}{\partial t} = \frac{2h(T_w - T)}{rp} + \frac{q_I C}{\pi r^2 \rho} \quad (16)$$

Finally, using equation (12) to eliminate ρ from equation (16) and rearranging yield

$$\frac{\partial T}{\partial t} = \frac{2hZRT}{rMpc_p} (T_w - T) - u \frac{\partial T}{\partial x} + \frac{RTZ_1}{JMPc_p} \frac{\partial P}{\partial t} + \frac{RTZq_I C}{\pi r^2 Mpc_p} \quad (17)$$

Equation (17) is the form of the first law of thermodynamics to be used in the analysis. The quantities Z and Z_1 depend on the local gas temperature T and the tank pressure P . The specific heat c_p depends primarily on T . The heat-transfer coefficient h depends on T_w as well as T and P . The pressure P is a prescribed function of time. The heat-transfer rate per unit area to the internal hardware q_I must be known or estimated, and C is a function of the geometry of the internal hardware.

Tank wall heat transfer. - The heat-transfer equation at a point in the tank wall can be written

$$\frac{\partial T_w}{\partial t} = \frac{h}{l_w \rho_w c_w} (T - T_w) + \frac{q_o}{l_w \rho_w c_w} \quad (18)$$

where q_o is the rate of heat addition per unit area to the tank wall from outside the tank (q_o must be specified as a function of time).

Continuity equation. - The one-dimensional equation of continuity can be written (ref. 5, p. 182)

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \quad (19)$$

or, by using equation (2) and rearranging,

$$\frac{\partial u}{\partial x} = - \frac{1}{\rho} \frac{D\rho}{Dt} = \rho \frac{D}{Dt} \left(\frac{1}{\rho} \right) \quad (20)$$

Since Z is a function of T and P , differentiating the equation of state (12) gives

$$\begin{aligned} \frac{D}{Dt} \left(\frac{1}{\rho} \right) &= \frac{R}{M} \frac{D}{Dt} \left(\frac{ZT}{P} \right) \\ &= \frac{R}{M} \left\{ \left[\frac{\partial}{\partial T} \left(\frac{ZT}{P} \right) \right]_P \frac{DT}{Dt} + \left[\frac{\partial}{\partial P} \left(\frac{ZT}{P} \right) \right]_T \frac{DP}{Dt} \right\} \\ &= \frac{R}{MP^2} \left[P \left(Z + T \frac{\partial Z}{\partial T} \right) \frac{DT}{Dt} - T \left(Z - P \frac{\partial Z}{\partial P} \right) \frac{DP}{Dt} \right] \\ &= \frac{R}{MP^2} \left(PZ_1 \frac{DT}{Dt} - TZ_2 \frac{DP}{Dt} \right) \end{aligned} \quad (21)$$

where Z_1 is given by equation (14) and

$$Z_2 \equiv Z - P \frac{\partial Z}{\partial P} \quad (22)$$

(In the special case of a perfect gas $Z = Z_1 = Z_2 = 1$.) Multiplying both sides of equation (21) by ρ and using equation (12) give

$$\rho \frac{D}{Dt} \left(\frac{1}{\rho} \right) = \frac{1}{ZTP} \left(PZ_1 \frac{DT}{Dt} - TZ_2 \frac{DP}{Dt} \right) = \frac{Z_1}{Z} \frac{1}{T} \frac{DT}{Dt} - \frac{Z_2}{Z} \frac{1}{P} \frac{DP}{Dt} \quad (23)$$

By substitution from equation (23), equation (20) becomes

$$\frac{\partial u}{\partial x} = \frac{Z_1}{ZT} \frac{DT}{Dt} - \frac{Z_2}{ZP} \frac{DP}{Dt} = \frac{Z_1}{ZT} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) - \frac{Z_2}{ZP} \frac{\partial P}{\partial t} \quad (24)$$

An alternate form of this equation, which will be used in the numerical solution only at time zero, is obtained by substituting from equation (17) to get

$$\frac{\partial u}{\partial x} = \frac{2hZ_1R}{rMPc_p} (T_w - T) + \left(\frac{RZ_1^2}{JMPZc_p} - \frac{Z_2}{ZP} \right) \frac{\partial P}{\partial t} + \frac{RZ_1q_I C}{\pi r^2 MPc_p} \quad (25)$$

Equations (17), (18), and (24) are the differential equations that describe the problem within the framework of assumptions (1) to (8).

Initial and boundary conditions. - To determine a unique solution of equations (17), (18), and (25), some further conditions must be specified. At time $t = 0$, the beginning of outflow, the gas and wall temperatures must be prescribed. In addition to these initial conditions, the following boundary conditions must be known: the variation with time of the inlet gas temperature, the outlet flow rate (which determines the velocity of the liquid surface), and the wall and gas temperatures at the liquid surface.

The boundary conditions necessary are commonly prescribed in the design. The initial conditions must be estimated or determined from a separate calculation or experiment.

Numerical Solution

The equations contained in the preceding section are so complex that a numerical solution is indicated. The numerical method to be presented here proceeds in the following general way. First, the differential equations involved are approximated by algebraic equations at a discrete set of points (in space and time) called net points. Second, the resulting set of algebraic equations is solved, and values of T , T_w , and u are obtained that approximate the solution of the differential equations at the net points. Ordinarily the solution of the algebraic equations becomes a better approximation to the true solution as the distance between net points is decreased.

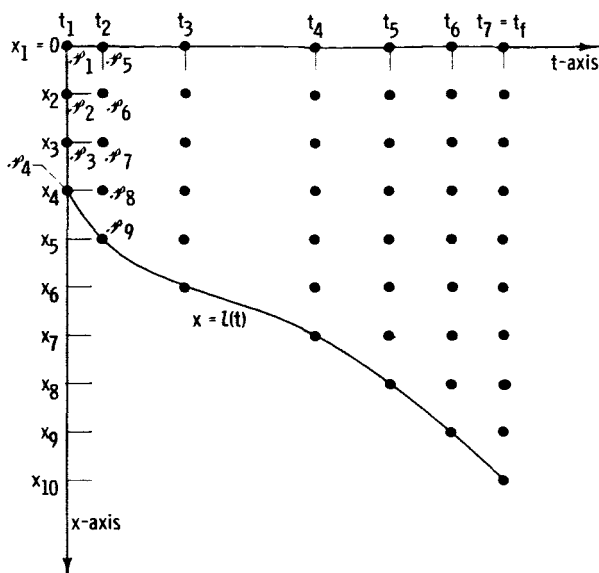


Figure 2. - Schematic drawing of space-time plane for variable outflow rate showing net points for $N = 4$.

P_5 , P_6 , P_7 , and P_8 . In addition, a net point is placed at the new location of the liquid surface, point P_9 in figure 2. A similar procedure is followed at each succeeding net value of time.

It was decided to keep the distance between net points constant in the x-direction. This was done by choosing each time step in such a way that the liquid surface advances exactly the distance of one net spacing in the x-direction during that time step. That is, Δt is chosen such that

$$\Delta t(t) = \frac{\Delta x}{u(l, t)} \quad (26)$$

For constant outflow rates, therefore, the time step is constant. In general, however, Δt may vary with time, as illustrated in figure 2.

It follows that the entire set of net points is determined by the choice of N (the number of net points that divide the initial ullage) and equation (26). The choice of a suitable value of N (which determines Δx) is largely a matter of experience. Some indication of the accuracy of a solution can be obtained by rerunning the problem with a finer net. If the results are substantially changed by the addition of more net points, the first net was too coarse. In that case, a third and finer net should be used in the same way.

Approximating equations. - Approximations of the differential equations (17), (18), and (24) can be made in many ways. General methods for approximating differential equations are given, for example, in references 6 to 8. Some of the approximations that were tried for equations (17), (18), and (24) proved to be unstable in the sense that solutions of the approximate equations oscillated with rapidly increasing amplitude at successive time steps and the temperatures went outside the range of the computing machine. This is a

Net points. - The region of the x,t-plane in which a solution is sought is shown schematically in figure 2. The boundaries $t = t_1 = 0$ and $t = t_f$ are the times corresponding to the beginning and end of outflow. The boundary $x = 0$ is the top of the cylindrical section of the tank (fig. 1), and the boundary $x = l(t)$ is the location of the liquid surface at time t . The latter boundary is a straight line when the outflow rate is constant with time.

At time $t = t_1 = 0$ a set of N net points is equally spaced in the x-direction, for example, the points P_1 , P_2 , P_3 , and P_4 of figure 2. At the next net value of time, $t = t_2$ in figure 2, net points are located at each value of x that was a net value of time $t = t_1$, for example, points

commonly occurring difficulty in the numerical solution of heat-transfer problems (discussed, e.g., in ref. 9). The approximate equations to be presented here have been used over a very wide range of the design parameters, and they appear to be stable.

Denote the net values of time by t_1, t_2, t_3 , etc., starting with $t_1 = 0$ (see fig. 2). Similarly denote the net values of x by x_1, x_2, x_3 , etc., starting with $x_1 = 0$. The approximating equations will be written at a typical net point (x_i, t_j) , not on a boundary.

The following notation is used for any dependent variable $f(x, t)$:

$$f_i \equiv f(x_i, t_{j-1})$$

$$f'_i \equiv f(x_i, t_j)$$

Thus, a subscript refers to the space variable x , and the prime refers to a step forward in time.

With this notation the algebraic equation that approximates equation (24) at a net point (x_i, t_j) is

$$\frac{u'_{i+1} - u'_i}{\Delta x} = \left(\frac{Z_1}{Z}\right)'_i \frac{1}{T'_i} \left(\frac{T'_i - T_i}{\Delta t} + u'_i \frac{T'_{i+1} - T'_i}{\Delta x}\right) - \left(\frac{Z_2}{Z}\right)'_i \frac{1}{P'} \frac{P' - P}{\Delta t}$$

where

$$\Delta t = t_j - t_{j-1}$$

Rearranging gives

$$u'_i = \frac{\left[T'_i u'_{i+1} - \left(\frac{Z_1}{Z}\right)'_i \frac{\Delta x}{\Delta t} (T'_i - T_i) + \left(\frac{Z_2}{Z}\right)'_i \frac{T'_i}{P'} \frac{\Delta x}{\Delta t} (P' - P) \right]}{\left[T'_i + \left(\frac{Z_1}{Z}\right)'_i (T'_{i+1} - T'_i) \right]} \quad (27)$$

Next, differential equation (17) is approximated at the net point (x_i, t_j) by

$$\begin{aligned} \frac{T'_i - T_i}{\Delta t} = & \left(\frac{2hZR}{rMPC_p} \right)^*_i (T'_{w,i} - T'_i) T'_i - u'_i \frac{T'_i - T'_{i-1}}{\Delta x} \\ & + \left(\frac{RZ_1}{JMPc_p} \frac{\partial P}{\partial t} + \frac{RZq_I}{\pi r^2 MPC_p} \right)^*_i T'_i \quad (28) \end{aligned}$$

The asterisk on a quantity indicates that the quantity is either to be evaluated at $t = t_{j-1}$ or else at $t = t_j$ by a process of iteration to be described.

Equation (18) is approximated at (x_i, t_j) by

$$\frac{T'_{w,i} - T_{w,i}}{\Delta t} = \left(\frac{h}{\lambda_w \rho_w c_w} \right)_i^* (T'_i - T'_{w,i}) + \left(\frac{q_o}{\lambda_w \rho_w c_w} \right)_i^* \quad (29)$$

Solving equation (29) for $T'_{w,i}$ gives

$$T'_{w,i} = \frac{1}{1 + \left(\frac{h \Delta t}{\lambda_w \rho_w c_w} \right)_i^*} \left[T_{w,i} + \left(\frac{h \Delta t}{\lambda_w \rho_w c_w} \right)_i^* T'_i + \left(\frac{q_o \Delta t}{\lambda_w \rho_w c_w} \right)_i^* \right] \quad (30)$$

Substituting $T'_{w,i}$ from equation (30) into equation (28) and rearranging give

$$(T'_i)^2 + \left\{ \alpha_i^* \left[1 + u_i^* \frac{\Delta t}{\Delta x} - \left(\frac{RZ_1}{JMPc_p} \frac{\partial P}{\partial t} \Delta t + \frac{RZq_I \Delta t}{\pi r^2 MPc_p} \right)_i^* \right] - T_{w,i} - \left(\frac{q_o \Delta t}{\lambda_w \rho_w c_w} \right)_i^* \right\} T'_i - \alpha_i^* \left(u_i^* \frac{\Delta t}{\Delta x} T'_{i-1} + T_i \right) = 0 \quad (31)$$

where

$$\alpha_i^* \equiv \frac{1 + \left(\frac{h \Delta t}{\lambda_w \rho_w c_w} \right)_i^*}{\left(\frac{2hRZ \Delta t}{rMPc_p} \right)_i^*} \quad (32)$$

Equation (31) is quadratic in T'_i .

Finally, equation (25) is approximated at net points (x_i, t_1) by

$$\frac{u_{i+1} - u_i}{\Delta x} = \frac{1}{2} \left[\left(\frac{2hZ_1 R}{rMPc_p} \right)_i + \left(\frac{2hZ_1 R}{rMPc_p} \right)_{i+1} \right] \left(\frac{T_{w,i} + T_{w,i+1}}{2} - \frac{T_i + T_{i+1}}{2} \right) + \frac{1}{2} \left\{ \left[\left(\frac{RZ_1^2}{JMPZc_p} - \frac{Z_2}{ZP} \right) \frac{\partial P}{\partial t} + \frac{RZ_1 q_I C}{\pi r^2 MPc_p} \right]_i + \left[\left(\frac{RZ_1^2}{JMPZc_p} - \frac{Z_2}{ZP} \right) \frac{\partial P}{\partial t} + \frac{RZ_1 q_I C}{\pi r^2 MPc_p} \right]_{i+1} \right\} \quad (33)$$

As described in the next section, equations (27), (30), (31), and (33), along with the prescribed initial and boundary conditions, determine the numerical solution.

Method of solution. - The following quantities must be supplied in order for the solution to proceed:

- (1) At time $t = 0$ the values of gas temperature T and wall temperature T_w as functions of x (initial conditions)
- (2) On the boundary $x = 0$ the value of inlet gas temperature T as a function of time (boundary condition)
- (3) At the liquid surface $x = l(t)$ the value of gas temperature T , wall temperature T_w , and velocity u as functions of time (boundary conditions)
- (4) Tank pressure P , outside heating rate q_o , inside heating rate q_I (along with the geometric factor C), and outflow rate as functions of time
- (5) Constant value of heat-transfer coefficient h , or an equation by which h can be calculated at each net point from values of T , T_w , and P
- (6) Tank radius r , initial ullage length $l(0)$, and wall thickness l_w
- (7) Tank wall material properties: density ρ_w and specific heat $c_w(T_w)$
- (8) Pressurizing gas properties: molecular weight M , specific heat $c_p(T)$, and compressibility factor $Z(P, T)$
- (9) Total time for run
- (10) Number of net points N to be used in initial ullage space

The solution proceeds in the following way. At time $t = t_1 = 0$ the right side of equation (33) can be evaluated from known quantities. The term Δx is known from $\Delta x = l(0)/(N - 1)$. Furthermore, the velocity u is known at the liquid surface (e.g., at point \mathcal{P}_4 in fig. 2). Then, equation (33) can be solved successively for u at points \mathcal{P}_3 , \mathcal{P}_2 , and \mathcal{P}_1 .

Having obtained values of u at each net point at time $t = t_1 = 0$, attention is turned to equation (31), which is quadratic in T'_i . If T'_i is to be the temperature at point \mathcal{P}_6 , then all the temperatures occurring in the coefficients of the quadratic equation, except T'_{i-1} , are known. But T'_{i-1} is the temperature at \mathcal{P}_5 , so it is known from the boundary conditions. The quantities marked with an asterisk would, if practicable, be evaluated at time $t = t_2$. As a first approximation, however, these quantities are evaluated at $t = t_1 = 0$. With this done, the coefficients of the quadratic equation (31)

are known, and the equation can be solved for T at ρ_6 . With T known at ρ_6 the equation can be applied again, T'_1 being the temperature at ρ_7 . This process continues giving successively all the gas temperatures down through ρ_8 at the new net time. At ρ_9 , T'_1 is known from the boundary conditions.

The gas temperatures are then known at all the net points at time $t = t_2$. In a similar way, equation (30) gives all the wall temperatures at the new net time. Note that equation (30) is used for the wall temperature at ρ_5 since this is not given as a boundary condition.

With T and T_w known at the new time $t = t_2$ equation (27) can be applied to give values of u'_1 at $t = t_2$. Note that equation (33) is used only at $t = t_1 = 0$.

Recall that the quantities in equation (31) marked with an asterisk were evaluated at time $t = t_1 = 0$ when that equation was used. Now these quantities can be reevaluated at time $t = t_2$ based on the values of T , T_w , and u just calculated at $t = t_2$. With these new values of the coefficients, the temperatures T'_1 at time $t = t_2$ can be recomputed from equation (31). Similarly, the temperatures $T'_{w,i}$ at time $t = t_2$ can be recomputed from equation (30). Finally, the velocities u'_1 at $t = t_2$ can be recomputed at $t = t_2$ from equation (27). This iteration process can be carried out as many times as desired before proceeding with the solution. In practice the iteration has not contributed significantly to improve the accuracy. No iteration is used in the examples presented in the section COMPARISON OF CALCULATED AND EXPERIMENTAL RESULTS.

With the values of T , T_w , and u now known at $t = t_2$ the solution can proceed to $t = t_3$. It is clear that the order of operations will be the same in going from t_1 to t_2 and in going from t_2 to t_3 . Thus, the solution proceeds in this way from one net time to the next until the problem is finished.

As a part of the solution just described the inlet gas velocity $u(0,t)$ is found at each net time. These velocities can be used to calculate the mass flow rate of the pressurizing gas into the tank. Alternately, the mass flow rate can be found by integrating the gas density in the tank at each net time. Both methods of computing mass flow rate were used, and the values of the two were compared as an indication of the adequacy of the net size.

COMPARISON OF CALCULATED AND EXPERIMENTAL RESULTS

A number of examples, corresponding to experiments previously conducted at the Lewis Research Center (by the procedure described in ref. 1) and others at the Lockheed-Georgia Company (ref. 2), were calculated by the method described in this report. The calculated pressurant-mass requirements are compared in this section with the experimental values. In addition, for the Lewis examples, the calculated and experimental temperature distributions are also compared.

TABLE I. - EXPERIMENTAL VALUES FOR LEWIS EXAMPLES

Example	Tank pressure, lb sq in.	Out-flow rate, cu ft sec	Time of out-flow, sec	Initial ullage depth, ft	Heat to internal hardware, q_{TC} , Btu (ft)(sec)	Gas temperature at interface, $^{\circ}R$	Wall temperature at interface, $^{\circ}R$	Initial inlet gas temperature, $^{\circ}R$	Initial inlet wall temperature, $^{\circ}R$	Pressurizing gas	Heat-transfer coefficient Btu (sq ft)(hr)($^{\circ}R$)
1	160	0.0669	350	0.525	0.334	57	46	488	206	H ₂	13.75
2	161	.2375	93	.467	1.462	57	46	484	210	H ₂	12.25
3	57	.0780	284	.483	.296	47	46	373	170	H ₂	7.09
4	58	.2238	101	.375	1.213	47	46	398	157	H ₂	6.67
5	164	.2340	95	.583	.628	57	46	395	194	H ₂	11.34
6	40	.2550	88	.483	1.577	44	46	385	176	H ₂	5.13
7	159	.0634	355	.658	.293	^a 57-46	46	521	207	He	12.31
8	159	.2598	90	.675	1.427	^a 57-48	46	524	161	He	11.15
9	159	.2365	100	.458	.323	^a 57-50	46	324	153	He	10.45
10	40	.0703	309	.442	.240	^a 44-35	46	347	148	He	5.25

^aTemperatures varied with time between two values shown.

The calculations were carried out on an IBM 7094-II computer. The average computing time for the 19 examples was 24 seconds per example. The number of net points ranged from 20 at time zero to 100 to 200 at the end of outflow.

Lewis Experiments

In reference 1 the authors describe some liquid-hydrogen expulsion experiments. The tank used was 27 inches in diameter and 89 inches in overall length with dished-head ends. It was constructed of 5/16-inch-thick 304 stainless steel plate. Heat leak was reduced to 40 Btu per hour per square foot by a vacuum jacket surrounding the entire tank.

The inlet gas diffuser was designed to direct the flow vertically downward with a flat velocity profile across the tank. The inner surface of the tank dome was insulated with a 1/2-inch layer of cork. No slosh baffles, ribs, or other features (except for instrumentation) were present to disturb the flow. The instrumentation, described in detail in reference 1, provided a significant heat sink in some of the runs.

In order to test the analytical model and method of solution described in this report, 10 runs were selected for analysis. These include pressurization with both hydrogen and helium, and they span a range of outlet flow rates, tank pressure levels, and inlet gas temperature variations. Some of the data, obtained in the experimental investigation described in reference 1, have not been published previously.

Table I gives values of the quantities (obtained from experimental data) that must be known, in addition to the tank material and dimensions, to carry out a solution. The initial gas and wall temperature distributions in the experiments were nearly linear, so that the values at each point in the ullage

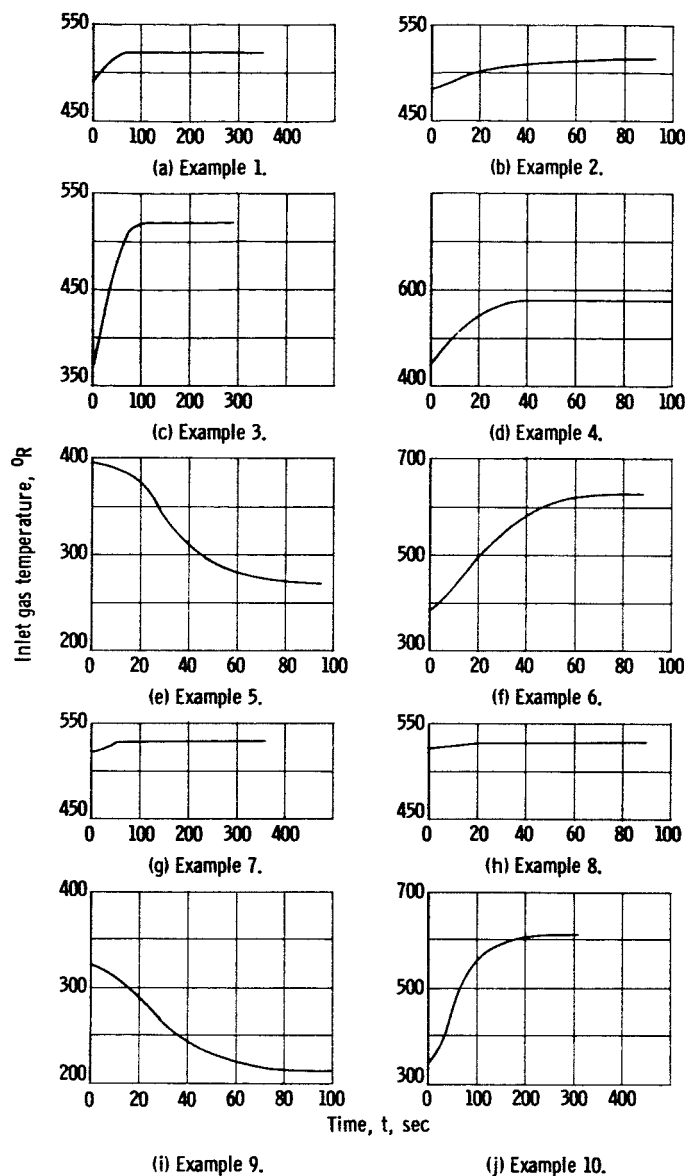


Figure 3. - Experimental inlet gas temperature distributions for Lewis examples.

computed from the PVT values of reference 10. For helium $Z = 1.0$ is adequate since the temperature does not get below liquid-hydrogen temperature.

are determined by the gas and wall temperatures given in table I. The constant heat-transfer coefficients shown in table I were obtained for each example by averaging the experimentally obtained local values over both space and time. The small heat-flow rate from the outside into the tank wall was neglected in all cases. The constant values of heat-flow rate to the internal hardware that appear in table I are averages based on estimated total heat flow to the hardware.

In addition to the quantities shown in table I, the variation with time of the inlet gas temperature must be prescribed. The temperatures used, in accordance with the assumptions of the analysis, are those measured at the top of the cylindrical part of the tank (at $x = 0$ in fig. 1). The experimental temperature-time variations for the 10 examples are given in figure 3. Both increasing and decreasing variations of inlet temperature are represented.

The values of specific heat used for the tank wall and for the hydrogen gas are given in table II. Values at atmospheric pressure were used in all cases. The specific heat of helium gas was taken to be constant, equal to 1.31 Btu per pound per $^{\circ}\text{R}$. The values of compressibility factor $Z(P,T)$ were

Figure 4 shows the calculated gas temperatures in the ullage at several times during outflow for example 1. Also shown are the measured gas temperatures at these same times. In figure 5 the same comparison is made for the tank wall temperatures. Figure 6 presents the gas and wall temperatures, both calculated and measured, at the end of outflow for each of the 10 examples. Although the agreement is generally good, the calculated temperatures are consistently high in the lower part of the tank.

TABLE II. - SPECIFIC HEAT FOR STAINLESS

STEEL AND HYDROGEN GAS

Stainless steel		Hydrogen gas	
Temperature, °R	Specific heat, Btu/(lb)(°R)	Temperature, °R	Specific heat, Btu/(lb)(°R)
46	0.0010	36.7	2.85
50	.0028	54	2.59
60	.0052	72	2.53
80	.0121	91	2.51
100	.0202	108	2.51
140	.0385	126	2.53
180	.0569	144	2.57
220	.0710	162	2.62
260	.0821	180	2.68
300	.0906	216	2.82
360	.0990	288	3.07
420	.1048	360	3.24
500	.1110	468	3.38
600	.1175	540	3.42

A comparison of measured and calculated total-pressurant-mass requirements is given in table III. The average deviation of calculated values from experimental values is 5.1 percent; the maximum deviation is 12.0 percent.

Lockheed-Georgia Company

Experiments

Reference 2 reports the results of hydrogen-expulsion experiments using a 40-inch-diameter tank, 100 inches in overall length. The tank was made of stainless steel 0.090 inch thick. The test tank was enclosed in a 60-inch-diameter vacuum-tight carbon steel tank. A gas diffuser was located in the top of the tank, and an antivortex baffle was in the bottom. Perforated conical slosh baffles were located at various axial positions. A probe was located on the tank centerline, with resistors for measuring gas and liquid temperature. Thermocouples were welded to the tank wall.

Tests are reported in reference 2 for various values of inlet temperature and initial ullage. Helium was used for pressurization in one case. Sloshing of the liquid was induced in all but one case.

Table IV gives the necessary experimental input data for the analysis as reported in reference 2. (Two experimental runs, in which leaks in the vacuum jacket occurred, are omitted from table IV.) In order to apply the cylindrical tank analysis to the experimental tank, which has a nearly spherical bottom, the following device was used: The actual tank was replaced for the purpose of the analysis by an equivalent cylindrical tank having the same diameter and volume.

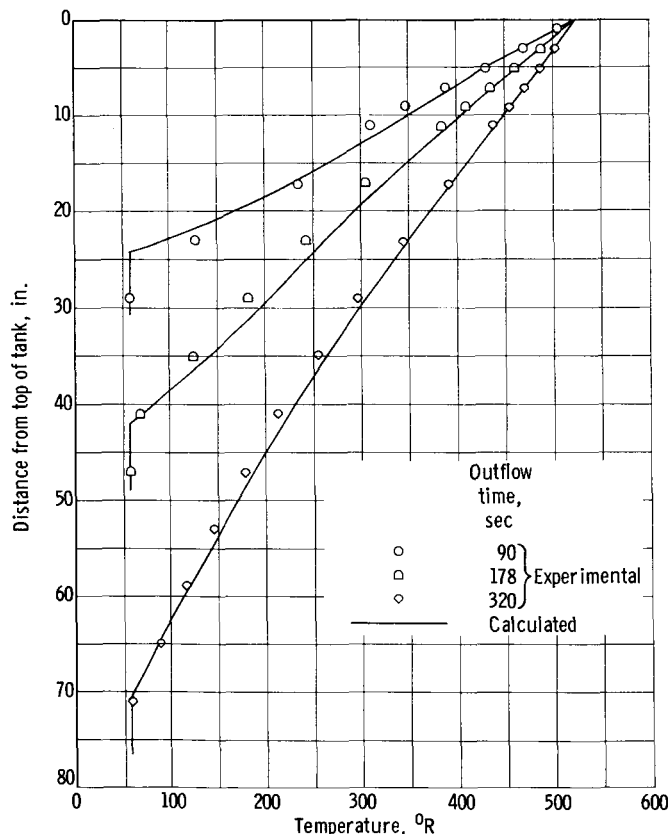


Figure 4. - Gas temperature distributions at three outflow times for example 1.

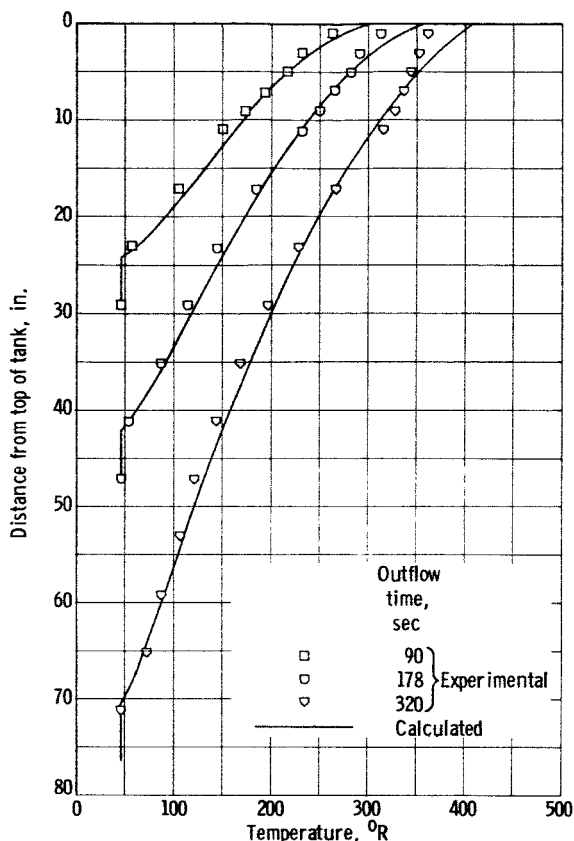


Figure 5. - Wall temperature distributions at three outflow times for example 1.

The equivalent cylindrical tank is about 6 inches shorter than the actual tank. This change in length causes a comparison of axial temperature distributions to have little meaning, so that only pressurant-mass requirements are compared.

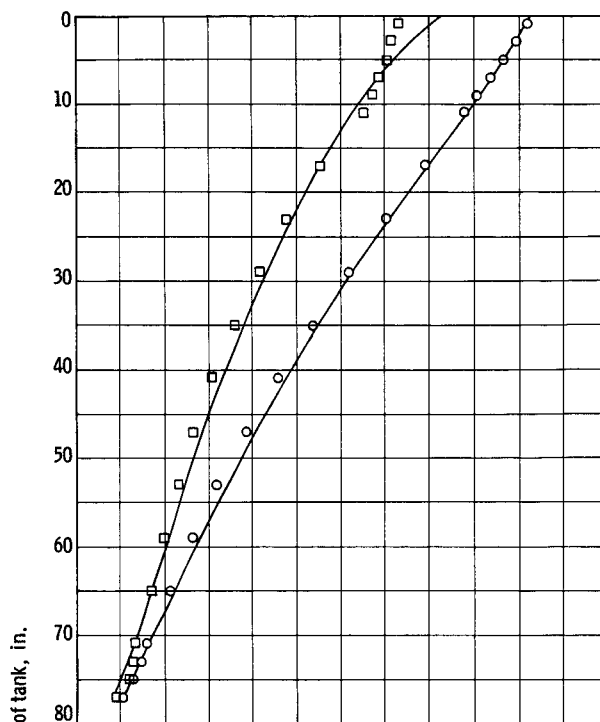
Table V gives the comparison of calculated and experimental values of total required pressurant mass. The average deviation of calculated results from experimental results is 4.4 percent. The maximum deviation is 7.7 percent. The good agreement, even with the occurrence of sloshing in the experiments, is partially attributed to the thin wall of the tank. The tank wall is thinner than in the Lewis experiments, and the effect of heat transfer compared to the effect of volume displacement on the pressurant-mass requirement is thereby smaller. The accuracy of any analytical method of predicting pressurant mass increases as the relative importance of the heat transfer in the problem decreases.

Effects of Internal Hardware, Real-Gas Properties, and Heat-Transfer Coefficient

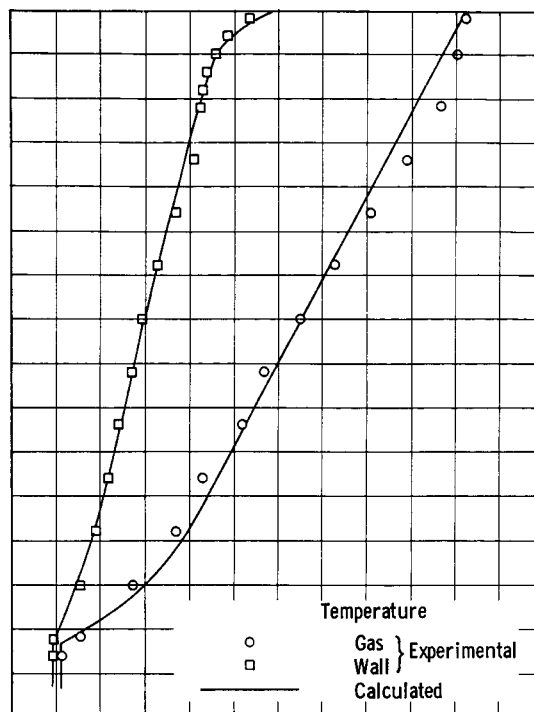
In order to examine the effect of neglecting heat transfer to the internal hardware, all of the Lewis experiments were rerun with $q_I = 0$. (The Lockheed experiments were calculated originally with $q_I = 0$ since no estimate for q_I was available.) The results in terms of pressurant-mass requirement are shown in table VI. With $q_I = 0$, for the Lewis runs, the average deviation of calculated from experimental values is 12.9 percent, compared to 5.1 percent obtained previously using values of q_I estimated from experimental data.

An examination was made also of the effect of using a perfect-gas equation of state ($Z(P,T) = 1.0$) in the calculations. The results again are shown in table VI. The average deviation from the experimental results when a perfect gas is used is 5.7 percent for the Lewis cases and 3.2 percent for the Lockheed cases compared with 5.1 and 4.4 percent, respectively, obtained previously for a real gas.

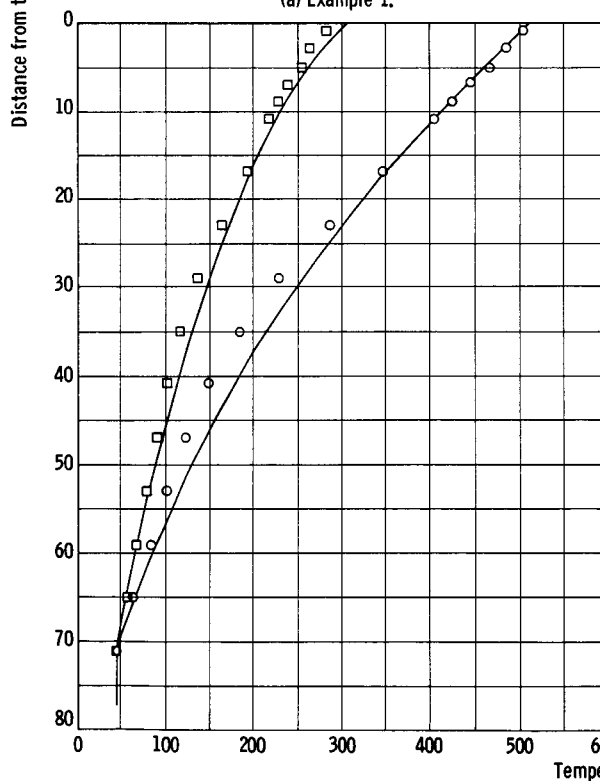
Finally, the experimental average values of heat-transfer coefficient were replaced in all the calculations by values computed locally from the free-convection formula (ref. 11, p. 172)



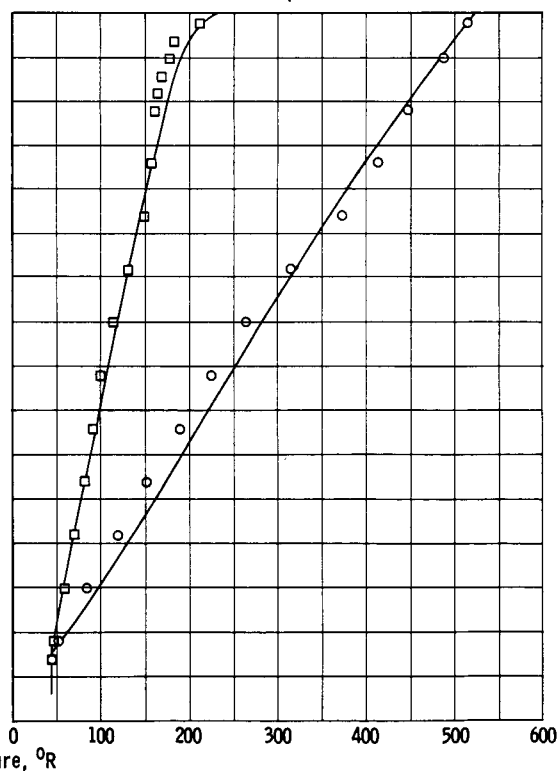
(a) Example 1.



(b) Example 2.

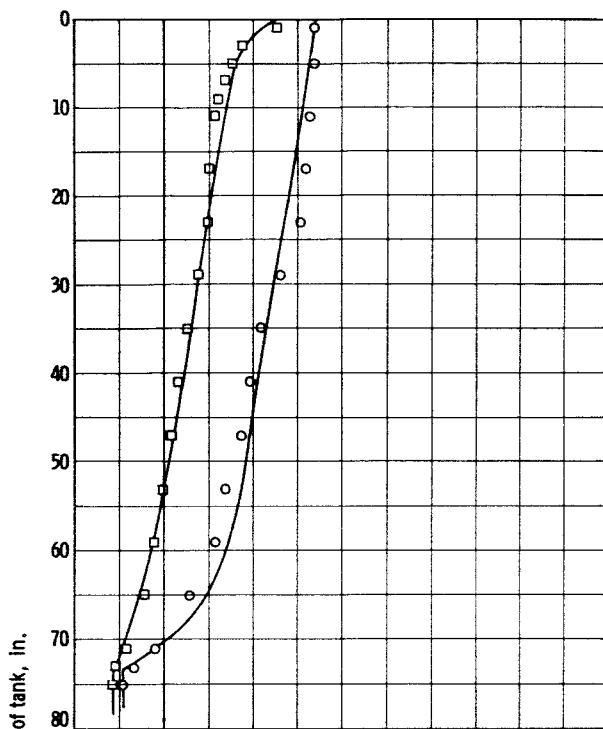


(c) Example 3.

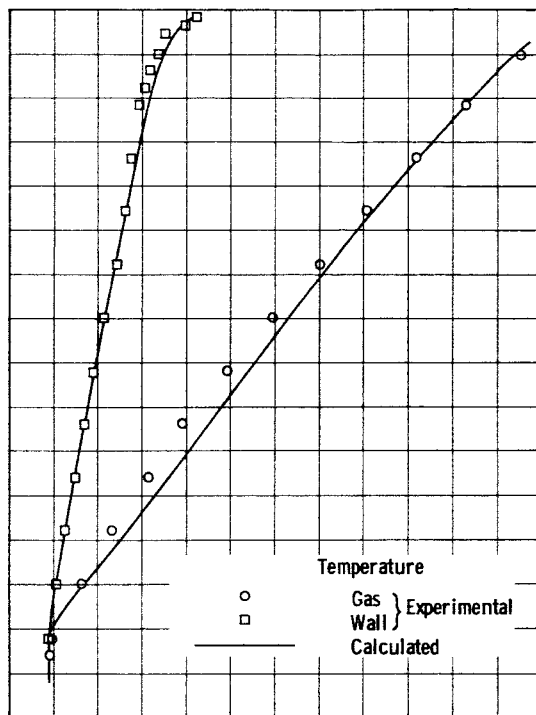


(d) Example 4.

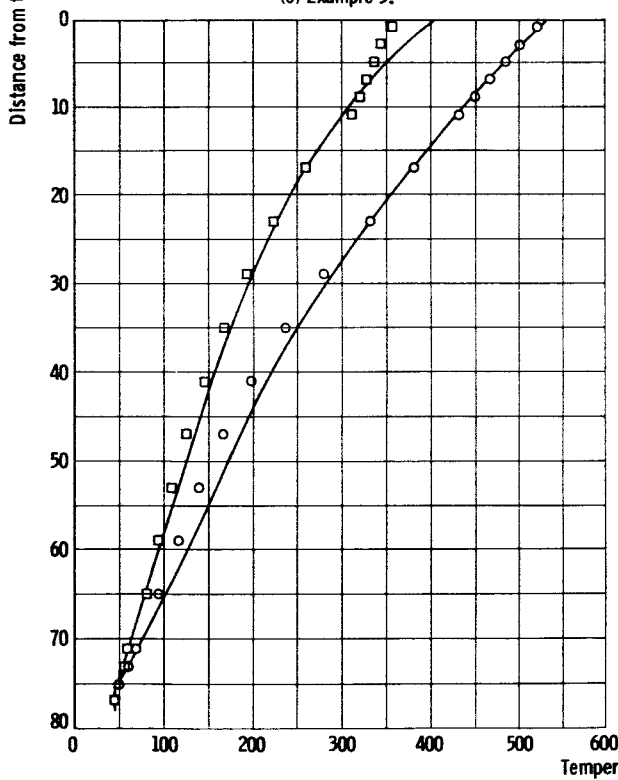
Figure 6. - Comparison of calculated and experimental gas and wall temperatures at end of outflow.



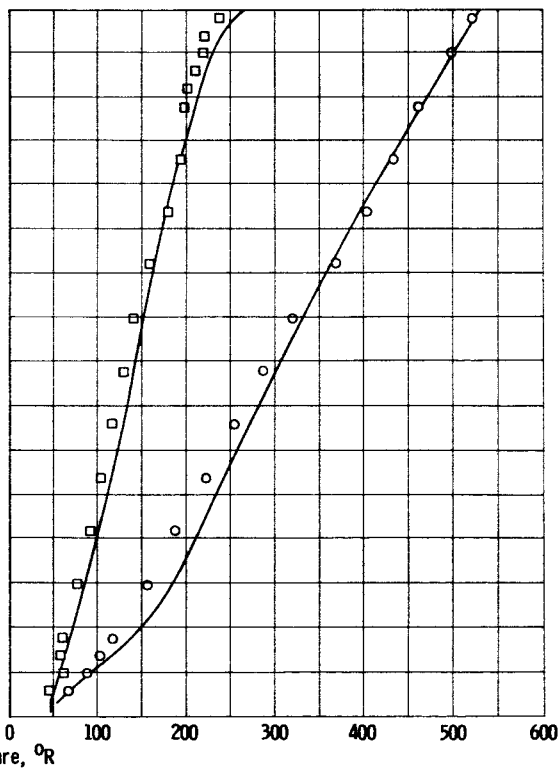
(e) Example 5.



(f) Example 6.

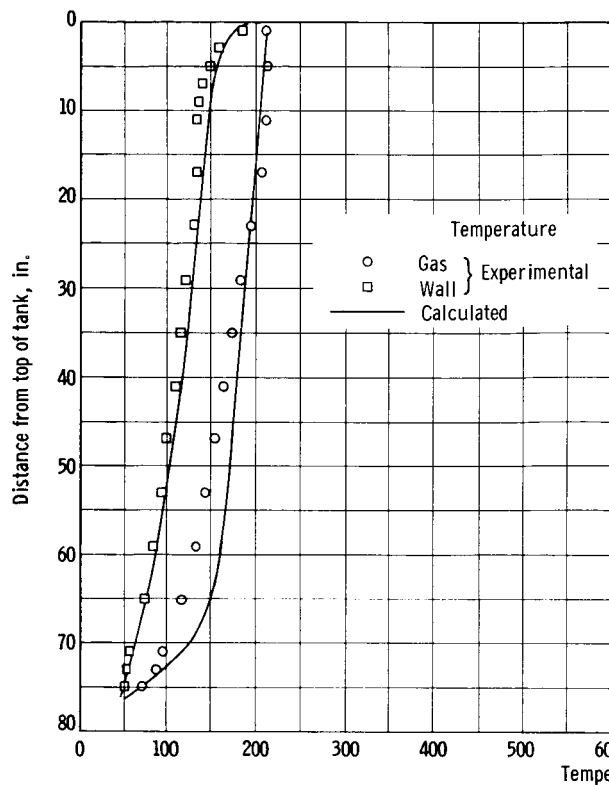


(g) Example 7.

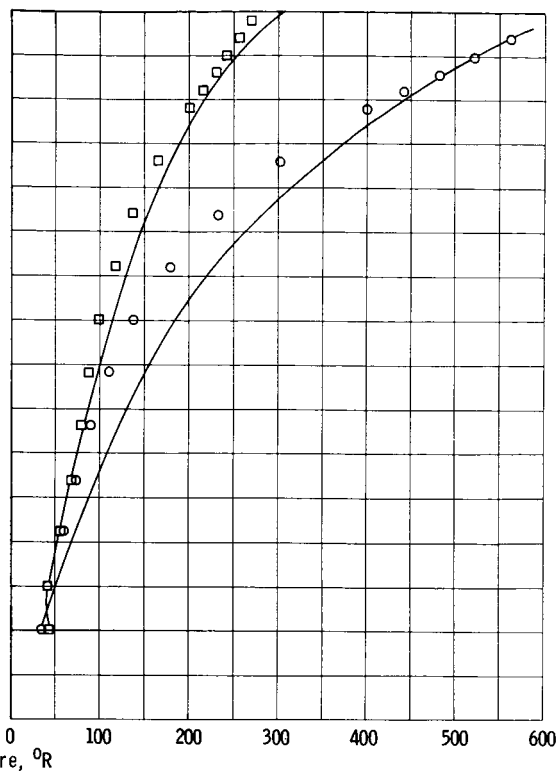


(h) Example 8.

Figure 6. - Continued. Comparison of calculated and experimental gas and wall temperatures at end of outflow.



(i) Example 9.



(j) Example 10.

Figure 6. - Concluded. Comparison of calculated and experimental gas and wall temperatures at end of outflow.

TABLE III. - PRESSURIZATION MASS

REQUIREMENTS FOR LEWIS EXAMPLES

Example	Experi- mental mass, m_e , lb	Calcu- lated mass, m_c , lb	Deviation, $\frac{m_c - m_e}{m_e} \times 100$, percent
1	3.98	3.95	-0.75
2	2.72	2.60	-4.41
3	1.76	1.68	-4.54
4	1.24	1.27	2.42
5	3.76	3.51	-6.65
6	.83	.93	12.04
7	8.14	7.61	-6.51
8	5.59	5.57	-.36
9	9.24	8.48	-8.23
10	2.70	2.56	-5.18

$$h = 0.13 \frac{k}{l} (Gr \ Pr)^{0.333}$$

The average deviation from the experimental results is 7.4 percent for the Lewis cases and 4.9 percent for the Lockheed cases compared with 5.1 and 4.4 percent, respectively, obtained with an experimental heat-transfer coefficient.

TABLE IV. - EXPERIMENTAL VALUES FOR LOCKHEED EXAMPLES

Example	Tank pressure, lb/sq in.	Out-flow rate, cu ft/sec	Time of out-flow, sec	Initial ullage depth, ft	Heat to internal hardware, Btu/(ft)(sec)	Gas temperature at interface, °R	Wall temperature at interface, °R	Initial inlet gas temperature, °R	Initial inlet wall temperature, °R	Pressurizing gas	Heat-transfer coefficient, Btu/(sq ft)(hr)(°R)
		(a)		(b)	(c)						
11	45.5	0.672	89	0.876	↓	^d 45	^d 45	300	^d 300	H ₂	11.5
12	47.6	.560	103	1.141		45	45	520	^d 500	H ₂	^d 12.0
13	46.5	.511	120	.684		45	45	300	262	H ₂	11.3
14	46.5	.607	87	1.758		45	45	300	236	H ₂	12.0
15	45.5	.609	99	.825		45	45	300	218	He	12.1
16	47.0	.644	95	.709		45	45	300	243	H ₂	12.3
17	45.0	.530	111	1.008		45	45	300	300	H ₂	11.8
18	46.2	.632	97	.692		45	45	300	277	H ₂	11.7
19	45.5	.565	105	.943		45	45	300	262	H ₂	13.9

^aComputed from reported outflow time, tank volume, and percent initial ullage.

^bComputed from reported percent initial ullage by neglecting curvature of tank ends.

^cNo information available to estimate heat flow to internal hardware.

^dEstimated, not given in ref. 2.

TABLE VI. - COMPARISON OF PRESSURANT MASS REQUIREMENTS

TABLE V. - PRESSURIZATION MASS REQUIREMENTS FOR LOCKHEED EXAMPLES

Example	Experimental mass, m _e , lb	Calculated mass, m _c , lb	Deviation, $\frac{m_c - m_e}{m_e} \times 100$, percent
11	2.61	2.81	7.67
12	2.13	2.24	5.17
13	2.86	3.05	6.64
14	2.57	2.65	3.11
15	5.79	5.89	1.73
16	2.47	2.58	4.45
17	2.81	2.86	1.78
18	2.81	2.95	4.98
19	2.88	3.00	4.17

Example	Mass of pressurant, lb				
	Experimental	Previously calculated	For zero heat flow to interior hardware	From ideal gas law	From computed heat-transfer coefficient
1	3.98	3.95	3.70	3.79	4.07
2	2.72	2.60	2.36	2.55	2.90
3	1.76	1.68	1.50	1.64	1.79
4	1.24	1.27	1.05	1.25	1.43
5	3.76	3.51	3.31	3.45	3.81
6	.83	.93	.69	.91	1.06
7	8.14	7.61	7.18	7.61	7.81
8	5.59	5.57	5.03	5.57	6.17
9	9.24	8.48	8.19	8.48	9.04
10	2.70	2.56	2.25	2.56	2.76
11	2.61	2.81	----	2.77	2.61
12	2.13	2.24	----	2.21	1.99
13	2.86	3.05	----	3.01	2.84
14	2.57	2.65	----	2.61	2.43
15	5.79	5.89	----	5.89	5.42
16	2.47	2.58	----	2.56	2.35
17	2.81	2.86	----	2.82	2.63
18	2.81	2.95	----	2.91	2.73
19	2.88	3.00	----	2.96	2.70

CONCLUDING REMARKS

The preceding results indicate a good agreement between predicted pressurant requirements and experimental values for both Lewis and Lockheed experiments. In addition, for the Lewis experiments, the wall and gas temperatures showed good agreement. This may indicate that the analysis can be successfully used to investigate the effect of various parameters in the tank pressurization problem.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, October 28, 1964

APPENDIX - SYMBOLS

C	effective perimeter of interior hardware, ft
c_p	gas specific heat, Btu/(lb)(°R)
c_w	wall specific heat, Btu/(lb)(°R)
e	specific internal energy, Btu/lb
Gr	Grashof number
h	heat-transfer coefficient, Btu/(sq ft)(sec)(°R)
i	specific enthalpy, Btu/lb
J	mechanical equivalent of heat, ft-lb/Btu
k	thermal conductivity, Btu/(ft)(sec)(°R)
l	x-coordinate of interface, ft
\bar{l}	characteristic length, $l + r$, ft
l_w	wall thickness, ft
M	molecular weight
m_c	calculated pressurant mass, lb
m_e	experimental pressurant mass, lb
N	number of net points at time zero
P	pressure, lb/sq ft
Pr	Prandtl number
Q	specific heat transfer rate from gas, Btu/(lb)(sec)
Q_I	specific heat transfer rate from gas to interior hardware, Btu/(lb)(sec)
Q_w	specific heat transfer rate from gas to wall, Btu/(lb)(sec)
q_I	heat transfer rate per unit area from interior hardware, Btu/(sq ft)(sec)
q_o	heat transfer rate to wall from outside, Btu/(sq ft)(sec)
R	universal gas constant, ft-lb/(lb)(mole)(°R)
r	tank radius, ft

T gas temperature, $^{\circ}\text{R}$
 T_w wall temperature, $^{\circ}\text{R}$
 t time, sec
 Δt time increment, sec
 t_f time at end of outflow, sec
 u gas velocity, ft/sec
 x coordinate in direction of tank axis, ft
 Δx space increment, ft
 Z compressibility factor
 Z_1 $Z + T \left(\frac{\partial Z}{\partial T} \right)_P$
 Z_2 $Z - P \left(\frac{\partial Z}{\partial P} \right)_T$
 α_i^* defined by eq. (32)
 ρ gas specific weight, lb/cu ft
 ρ_w wall specific weight, lb/cu ft

Subscripts:

i, j quantity evaluated at net point, x_i, t_j

Superscripts:

' next time step

* quantity to be evaluated at $t = t_{j-1}$ or at $t = t_j$

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